On Semantic Issues in Game-theoretic Rough Set Model

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Outline

Rough Sets

- Sets derived from imperfect, imprecise, and incomplete data may not be able to be precisely defined.
- Sets have to be approximated.
- Approximating a concept C with objects in U.
    - Lower approximation given by $\underline{apr}(C)\{x \in U | x \subset C\}$.
    - Upper approximation given by $\overline{apr}(C)\{x \in U | x \bigcap C \neq \phi\}$.
- The three regions defined by the approximations.
    - $POS(C) = \underline{apr}(C)$
    - $BND(C) = \overline{apr}(C) - \underline{apr}(C)$.
    - $NEG(C) = U - (POS(C) \bigcup BND(C))$.

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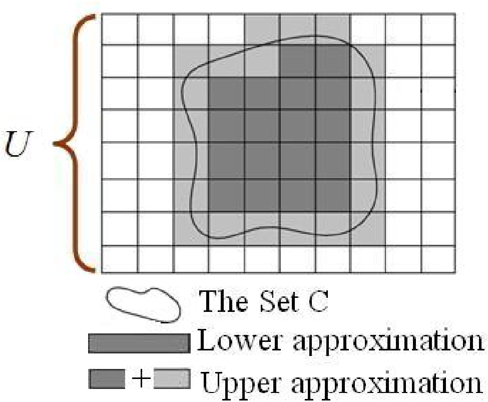
Rough Sets



Probabilistic Rough Sets

- Defining the approximations in terms of conditional probabilities and a pair of thresholds (Yao, 2008).
  - The (α, β) thresholds for determining the probabilistic rough set approximations given by,

$$\begin{aligned} \underline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{[x] \in U/E \mid Pr(C|[x]) \geq \alpha\}, \\ \overline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{[x] \in U/E \mid Pr(C|[x]) > \beta\}. \end{aligned} \quad (1)$$

  - Probabilistic positive, negative and boundary regions:

$$\begin{aligned} \mathrm{POS}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \geq \alpha\}, \\ \mathrm{NEG}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \leq \beta\}, \\ \mathrm{BND}_{(\alpha,\beta)}(C) &= \{x \in U \mid \beta < Pr(C|[x]) < \alpha\}. \end{aligned} \quad (2)$$

Yao, Y. Y., (2008). Probabilistic rough set approximations, International Journal of Approximate Reasoning, 49.

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Three-way Decisions with Probabilistic Rough Sets

- Three-way decisions are made according to the following rules.

$$\begin{aligned} &\text{Acceptance:} && \text{if } P(C|[x]) \geq \alpha, \\ &\text{Rejection:} && \text{if } P(C|[x]) \leq \beta, \text{ and} \\ &\text{Deferment:} && \text{if } \beta < P(C|[x]) < \alpha. \end{aligned} \tag{3}$$

- A major difficulty is the interpretation and determination of the (α, β) thresholds (Yao, 2011).

Yao, Y.Y., (2011). Two semantic issues in a probabilistic rough set model. Fundamenta Informaticae 108(3-4).

Determination of (α, β) Probabilistic Thresholds

- Realizing the determination of probabilistic thresholds as an optimization based on criterion C.

$$\arg \max_{(\alpha,\beta)} C(\alpha, \beta), \text{ where}$$

$$C(\alpha, \beta) = C_P(\alpha, \beta) + C_N(\alpha, \beta) + C_B(\alpha, \beta). \qquad (4)$$

- Many attempts have been made to determine the thresholds.
  - Optimization viewpoint (Jia et al., 2011),
  - Multi-view model(Li and Zhou, 2011),
  - Method using probabilistic model criteria (Liu et al., 2011),
  - Information-theoretic interpretation (Deng and Yao, 2012) ,
  - Game-theoretic framework (Herbert and Yao, 2011).
- We consider the game-theoretic rough set model.

Jia, X. Y., Li, W. W., Shang, L., Chen, J. J., (2011). An optimization viewpoint of DTRS model. In: (RSKT'11).
Li, H.X., Zhou, X.Z., (2011). Risk decision making based on DTRS... IJCIS 4,
Liu, D., Li, T.R., Ruan, D., (2011). Probabilistic model criteria with DTRS. Information Science 181.
Deng, X. F., Yao, Y. Y., (2012). An information-theoretic interpretation of thresholds in PRS. In: (RSCTC'12)
Herbert, J.P., Yao, J.T., 2011. Game-theoretic rough sets. Fundamenta Informaticae 108.

Game Theory

- Game theory is a core subject in decision sciences.
- The basic game components include.
  - Players.
  - Strategies.
  - Payoffs.
- A classical example in Game Theory: The prisoners dilemma.

		p_2	
		confess	don't confess
p_1	confess	p_1 serves 10 year, p_2 serves 10 years	p_1 serves 0 year p_2 serves 20 years
	don't confess	p_1 serves 20 year, p_2 serves 0 years	p_1 serves 1 year, p_2 serves 1 years

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A Formal Game Definition

- A game may be formally defined as a tuple $\{P, S, u\}$ (Brown and Shoham, 2008),
    - P is a finite set of n players, indexed by i,
    - $S = S_1 \times ... \times S_n$, where S_i is a finite set of strategies available to player i. Each vector $s = (s_1, s_2, ..., s_n) \in S$ is called a strategy profile where player i selects strategy s_i.
    - $u = (u_1, ..., u_n)$ where $u_i : S \longmapsto \Re$ is a real-valued utility or payoff function for player i.

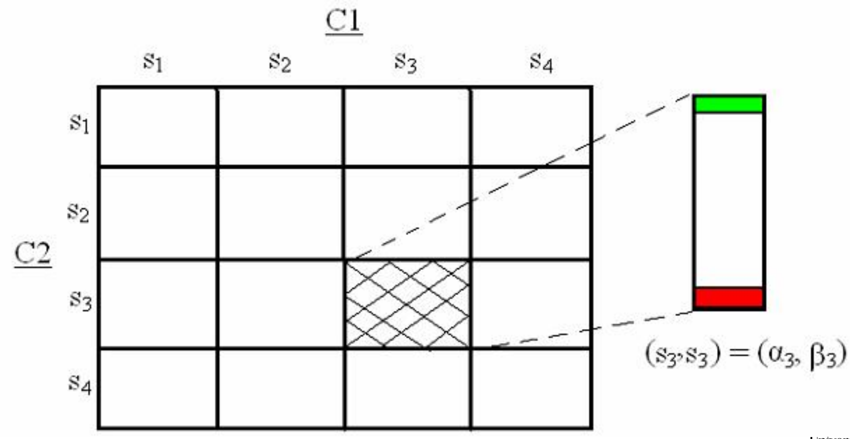
Brown L. K. and Shoham, Y., (2008). Essentials of Game Theory: A Concise Multidisciplinary Introduction.

Game-theoretic Rough Sets

- Utilizing a game-theoretic setting for analyzing rough sets.
- Determining the probabilistic thresholds to obtain the three regions and the implied three-way decisions.
- Current GTRS based formulations.
  - Game for improving classification ability (Herbert and Yao, 2011).
  - Game for obtaining effective rules (Azam and Yao, 2012).
  - Game for reducing region uncertainties (Azam and Yao, 2013).
  - Game for optimizing Gini Coefficient (Yan, 2011).

Herbert, J.P., Yao, J.T. (2011). Game-theoretic rough sets. Fundamenta Informaticae 108.
Azam, N., Yao, J. T., (2012). Multiple criteria decision analysis with GTRS. In: (RSKT'12).
Azam, N., Yao, J. T., (2013). Analyzing uncertainties of probabilistic rough set regions with GTRS. IJAR.
Yan, Z., (2013). Optimizing GINI coefficient of probabilistic rough set regions using GTRS. In: (CCECE13).

Game-theoretic Rough Sets



Semantic Issues in GTRS

- Interpreting the GTRS based game and its components.
  - Interpreting the players based on application requirements.
  - Understanding strategies based on threshold configuration levels.
  - Strategy profiles and their mappings to probabilistic thresholds.
- Meaning of determined thresholds based on a game outcome.

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Interpreting the GTRS based Game

- Implementing a game based on an application needs.
  - The needs may be represented in the form of multiple performance evaluating factors or criteria such as cost, risk, accuracy etc.
- A multi-objective optimization problem may be realized to meet these application needs.

$$\arg \min_{(\alpha,\beta)} C(\alpha,\beta), \text{ where}$$
$$C(\alpha,\beta) = (C_1(\alpha,\beta), C_2(\alpha,\beta), ..., C_n(\alpha,\beta)) \quad (5)$$

- The GTRS based game considers the above optimization as game-theoretic competition or cooperation among multiple criteria.

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Interpreting the Players

- Selecting the players to highlight different aspects of application specific needs.
- Example: considering an application which requires an improvement in the classification ability.
  - Accuracy represents one aspect of the requirement.
  - Precision represents another aspect.
- The players may compete or cooperate to reach these game objectives.

Interpreting the Strategies

- Considering strategies as different threshold modification levels.
- Using functions to represent startegies.
  - A strategy s_i of a player j that changes the thresholds, we may use functions to represent s_i as,

$$s_i = \{(f_i^j(\alpha), g_i^j(\beta)) \mid f_i^j(\alpha) = \alpha \pm c_1,\ g_i^j(\beta) = \beta \pm c_2)\} \quad (6)$$

    c_1, c_2 are the amount by which we modify the thresholds.
  - The threshold values calculated by the functions may be denoted by,

$$\begin{aligned} f_i^j(\alpha) &= \alpha \pm c_1 = \alpha_i^j \\ g_i^j(\beta) &= \beta \pm c_2 = \beta_i^j \end{aligned} \quad (7)$$

Mapping a Strategy to a Threshold Pair

- Associating a strategy with a threshold pair.
  - The strategy s_i of player j based on Eq.(6)-(7) can now be associated with (α_i^j, β_i^j).
- The functions (f_i^j, g_i^j) provides a mapping that maps each strategy s_i of player j to a threshold pair.

$$(f_i^j, g_i^j): s_i \longmapsto (D_\alpha, D_\beta), \tag{8}$$

  where $D_\alpha = D_\beta = [0, 1]$ are the domains of thresholds.
- In summary, each strategy leads to a threshold pair.

Interpreting the Strategy Profiles

- Strategy profiles are the possible combination of strategies in a game.
- Considering a special strategy profile $s = (s_1, s_2, ..., s_n)$, where player j plays s_j.
  - This may be represented in terms of functions defined for individual strategies in equation (6).

$$s = (s_1, s_2, ..., s_n) = ((f_1^1(\alpha), g_1^1(\beta)), ..., (f_n^n(\alpha), g_n^n(\beta))) \quad (9)$$

which leads to threshold pairs,

$$s = (s_1, s_2, ..., s_n) = ((\alpha_1^1, \beta_1^1), ..., (\alpha_n^n, \beta_n^n)) \qquad (10)$$

Mapping a Strategy Profile to a Threshold pair

- Realizing a strategy profile and its mapping to a threshold pair.
  - Additional functions may be used for this purpose.

$$s = (s_1, s_2, ..., s_n) = \{(H_s(\alpha_1^1, \alpha_2^2, ..., \alpha_n^n), O_s(\beta_1^1, \beta_2^2, ..., \beta_n^n))\},$$
$$\text{where} \quad H_s(\alpha_1^1, \alpha_2^2, ..., \alpha_n^n) = \alpha_s, O_s(\beta_1^1, \beta_2^2, ..., \beta_n^n) = \beta_s. \quad (11)$$

  - The strategy profile s is now associated with (α_s, β_s).
- The functions (H_s, O_s) maps a strategy profile s to another threshold pair,

$$(H_s, O_s): s \longmapsto (D_\alpha, D_\beta). \quad (12)$$

The GTRS based Game

- A GTRS based game has now the form $\{P, S, u\}$, where
  - $P =$ a finite set of n players considered as criteria for evaluating application specific requirements.
  - $S = S_1 \times ... \times S_n$, where S_j is a finite set of strategies available to player j. Each s_i of player j maps to a threshold pair by using functions f_i^j and g_i^j given by $(f_i^j(\alpha), g_i^j(\beta))$:$s_i \longmapsto (D_\alpha, D_\beta)$,
  - Each strategy profile of the form $s = (s_1, s_2, ..., s_n)$ also maps to a threshold pair given by (H_s, O_s):$s \longmapsto (D_\alpha, D_\beta)$.
  - $u = (u_1, ..., u_n)$ where $u_j : S \longmapsto \Re$ is a real-valued utility or payoff function for player j.

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Interpreting the Thresholds Determined by GTRS

- The utility of player j corresponding to the strategy profile s that maps to (α_s, β_s) is given by,

$$u_j(s) = u_j(\alpha_s, \beta_s) \tag{13}$$

- Let $s_{-j} = \{s_1, s_2, ..., s_{j-1}, s_{j+1}, ..., s_n\}$,
    - We may write $s = (s_j, s_{-j})$.
    - The utility of player j becomes $u_j(s) = u_j(s_j, s_{-j}) = u_j(\alpha_{(s_j,s_{-j})}, \beta_{(s_j,s_{-j})})$.

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The Game Outcome and the Determined Thresholds

- Interpreting the output or determined thresholds as solution concept of Nash equilibruim.
- Definition of determined thresholds with GTRS.

The GTRS determines a threshold pair that corresponds to a strategy profile $s = (s_1, s_2, ..., s_n) = (s_j, s_{-j})$ such that

$$u_j(\alpha_{(s_j, s_{-j})}, \beta_{(s_j, s_{-j})}) \geq u_j(\alpha_{(s_j', s_{-j})}, \beta_{(s_j', s_{-j})}), \quad \text{where } (s_j^{'} \in S_j \wedge s_j^{'} \neq s_j) \tag{14}$$

Interpreting Herbert and Yao, (2011) Formulation

- The objective was to obtain effective region sizes.
- A competitive game was considered between the probabilistic region parameters α and β.
- The set of players in the game $P = \{\alpha, \beta\}$.
- Three strategies were considered for player 1, i.e. α.
  - The strategy set of player 1 $= S_1 = \{s_1, s_2, s_3\}$, where
  - $s_1 =$ decrease α by 5%,
  - $s_2 =$ decrease α by 7%, and
  - $s_3 =$ decrease α by 15%,
- Similar strategies were defined for player β.

Herbert, J.P., Yao, J.T. (2011). Game-theoretic rough sets. Fundamenta Informaticae 108, 267-286.

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Interpreting the Strategies

- The strategies may be represented using equation 6.
- Considering the strategies of player α.

$$
\begin{aligned}
s_1 &= \{f_1^1(\alpha) = \alpha - c_1 \times \alpha = \alpha(1 - 0.05) = 0.95\alpha,\ g_1^1(\beta) = \beta\} \\
s_2 &= \{f_2^1(\alpha) = \alpha - c_2 \times \alpha = \alpha(1 - 0.07) = 0.93\alpha,\ g_2^1(\beta) = \beta\} \\
s_3 &= \{f_3^1(\alpha) = \alpha - c_3 \times \alpha = \alpha(1 - 0.15) = 0.85\alpha,\ g_3^1(\beta) = \beta\} \quad (15)
\end{aligned}
$$

- The corresponding threshold pairs are given by,

$$
\begin{aligned}
s_1 &= (\alpha_1^1, \beta_1^1) = (0.95\alpha, \beta) \\
s_2 &= (\alpha_2^1, \beta_2^1) = (0.93\alpha, \beta), \\
s_3 &= (\alpha_3^1, \beta_3^1) = (0.85\alpha, \beta). \qquad (16)
\end{aligned}
$$

- Similar interpretation applies to strategies of β.

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Interpreting the Strategy Profiles

- There were nine strategy profiles in this game.

$$S = S_1 \times S_2 = \{(s_1, s_1), (s_1, s_2), ..., (s_3, s_2), (s_3, s_3)\}. \quad (17)$$

- Considering the profile (s_1, s_1), we have

$$\begin{aligned}(s_1, s_1) = \{&H_{(s_1,s_1)}(\alpha_1^1, \alpha_1^2) = H_{(s_1,s_1)}(0.95\alpha, \alpha),\\ &O_{(s_1,s_1)}(\beta_1^1, \beta_1^2) = O_{(s_1,s_1)}(\beta, 1.05\beta)\}\end{aligned} \quad (18)$$

- The threshold pair corresponding to (s_1, s_1) was determined as,

$$H_{(s_1,s_1)}(\alpha_1^1, \alpha_1^2) = 0.95\alpha, \quad O_{(s_1,s_1)}(\beta_1^1, \beta_1^2) = 1.05\beta. \quad (19)$$

- Final threshold values may be determined using the Nash equilibrium solution as defined in Equation 14

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Conclusion

- Existing GTRS based formulations and approaches extended the applicability of the model.
- The differences in treatment of game components and determination of thresholds may lead to possible confusions and misinterpretation.
- We address some semantic issues related to the interpretation of game components and the determination of thresholds with GTRS.
- It is hoped that this will improve the understandability of GTRS.
  - Ultimately leading to more interesting applications.

Questions?