Formulating Three-way Decision Making with Game-theoretic Rough Sets

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Outline

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Three-way Decision Making

- Three-way decisions are common in every day life.
- For example, medical field
  - Doctor examining a patient for a possible disease.
  - A disease may be present or absent.
  - Based on evidence the doctor can decide weather to treat, don't treat or do further examination to reach a conclusion.
- Other examples may be found in spam email filtering, investment decisions and peer review process.

Three-way Decision Making

- The problem of three-way decisions [8]

Considering U as a finite nonempty set of objects and C as a set of criteria. The problem of threeway decisions is to divide U based on C into three disjoint regions, POS, NEG and BND called as positive, negative and boundary regions, respectively.

Rough Sets

- Sets derived from imperfect, imprecise, and incomplete data may not be able to be precisely defined.
- Sets have to be approximated.
- Approximating a concept C with objects in U.
  - Lower approximation given by $\underline{apr}(C)\{x \in U | x \subset C\}$.
  - Upper approximation given by $\overline{apr}(C)\{x \in U | x \bigcap C \neq \phi\}$.
- The three regions defined by the approximation
  - $POS(C) = \underline{apr}(C)$
  - $BND(C) = \overline{apr}(C) - \underline{apr}(C)$.
  - $NEG(C) = U - (POS(\overline{C}) \bigcup BND(C))$.

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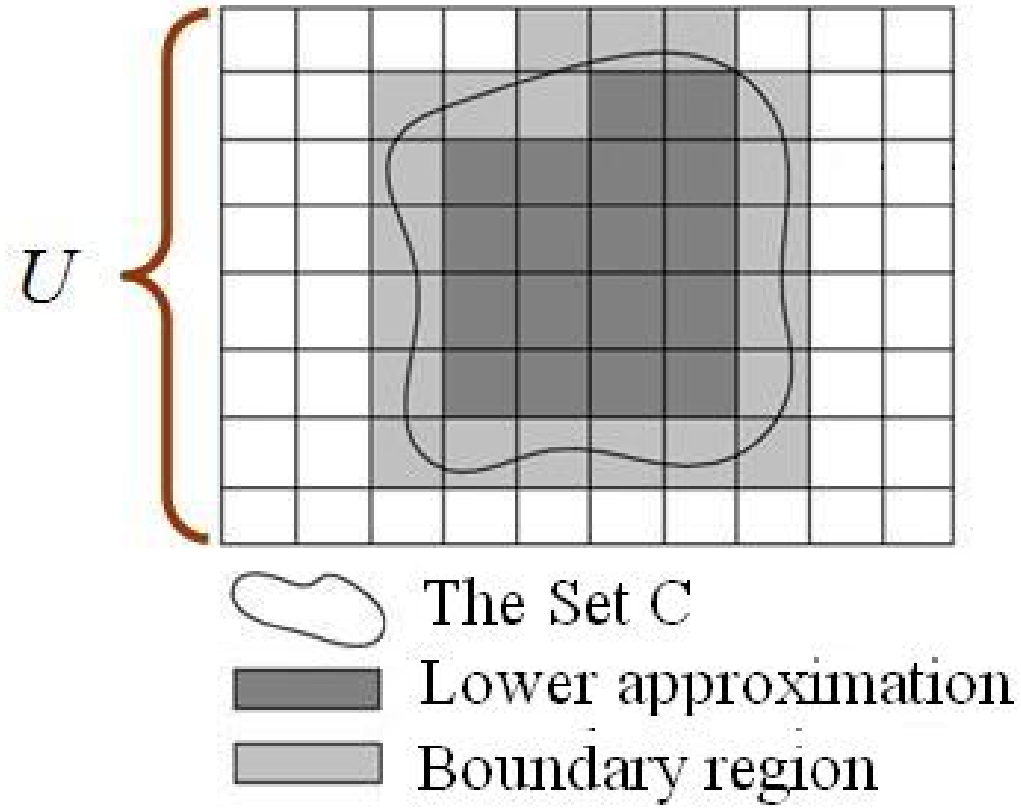
Rough Sets



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Probabilistic Rough Sets

- Defines the approximations in terms of conditional probabilities.
  - Introduces a pair of threshold denoted as (α, β) to determine the probabilistic rough set approximations given by,

$$\begin{aligned} \underline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{[x] \in U/E \mid Pr(C|[x]) \geq \alpha\}, \\ \overline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{[x] \in U/E \mid Pr(C|[x]) > \beta\}. \end{aligned} \quad (1)$$

  - Probabilistic positive, negative and boundary regions:

$$\begin{aligned} \mathrm{POS}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \geq \alpha\}, \\ \mathrm{NEG}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \leq \beta\}, \\ \mathrm{BND}_{(\alpha,\beta)}(C) &= \{x \in U \mid \beta < Pr(C|[x]) < \alpha\}. \end{aligned} \quad (2)$$

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Three-way Decisions with Probabilistic Rough Sets

- Three-way decisions are made according to the following rules.

$$\begin{aligned} &\text{Acceptance:} && \text{if } P(C|[x]) \geq \alpha, \\ &\text{Rejection:} && \text{if } P(C|[x]) \leq \beta, \text{ and} \\ &\text{Deferment:} && \text{if } \beta < P(C|[x]) < \alpha. \end{aligned} \tag{3}$$

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Determination of thresholds (α, β)

- Many attempts have been made to determine the thresholds.
  - Xiuyi Jia's [2] optimization viewpoint,
  - Huaxiong Li's [3] multi-view model,
  - Dun Liu's [4] method using probabilistic model criteria,
  - Deng and Yao [7] information-theoretic interpretation,
  - Yao and his colleagues [1, 5, 6] game-theoretic framework.

Game Theory

- Game theory is a core subject in decision sciences.
- The basic game components include.
  - Players.
  - Strategies.
  - Payoffs.
- A classical example in Game Theory: The prisoners dilemma.

		p_2	
		confess	don't confess
p_1	confess	p_1 serves 10 year, p_2 serves 10 years	p_1 serves 0 year p_2 serves 20 years
	don't confess	p_1 serves 20 year, p_2 serves 0 years	p_1 serves 1 year, p_2 serves 1 years

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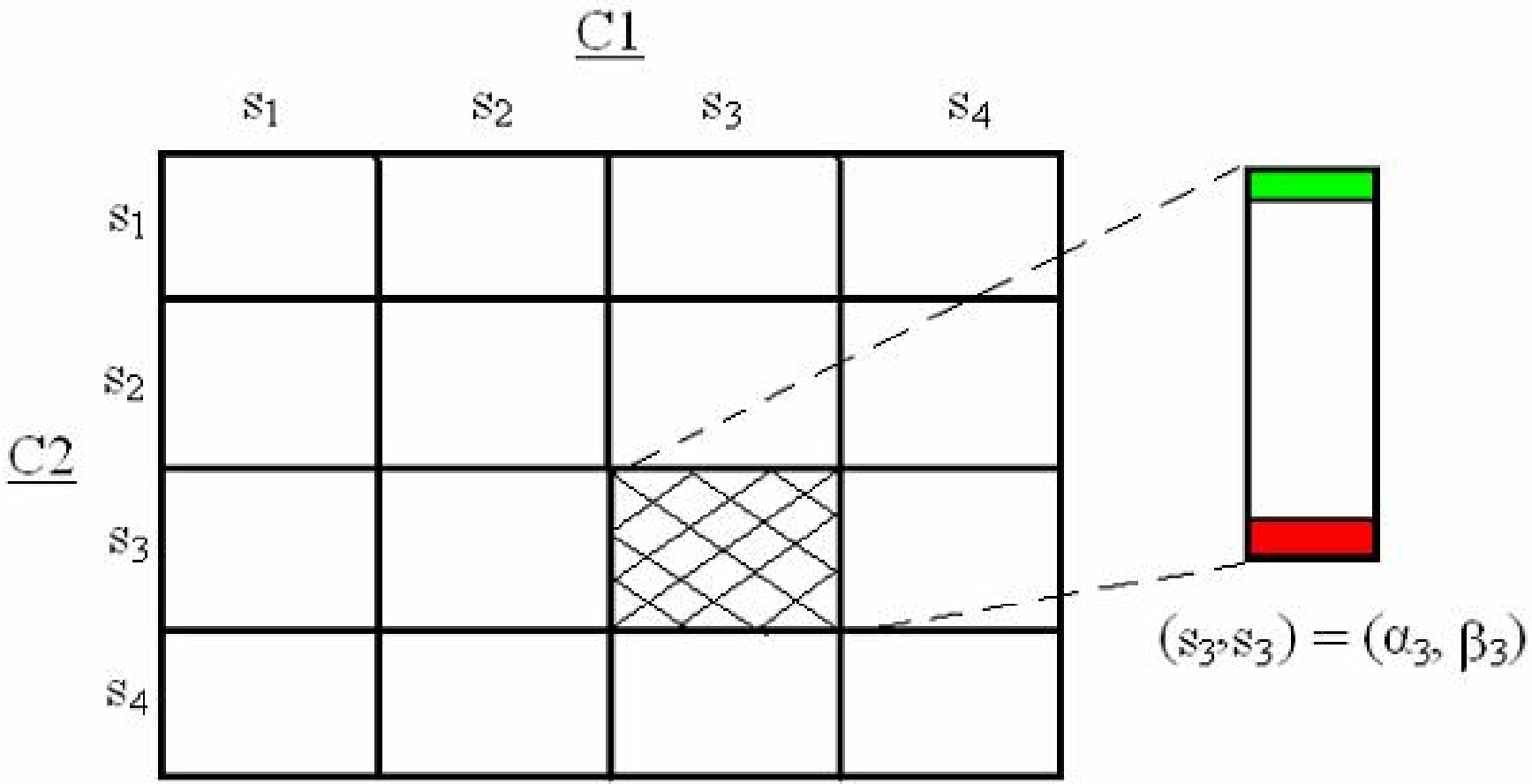
Game-theoretic Rough Sets

- Utilizing a game-theoretic setting for analyzing rough sets.
- Determining the probabilistic thresholds to obtain the three regions and the implied three-way decisions.
- Examples
  - Game of improving classification ability.
  - Game of for obtaining effective rules.
  - Game for reducing region uncertainties.

Game-theoretic Rough Sets



Game-theoretic Rough Sets

- In probabilistic rough sets the criterion of conditional probability of a concept is used to obtain three-way decisions.
- We try to incorporate multiple criteria in making three-way decisions.
- The GTRS can play a role here.

Formulation of Three-way Decisions with GTRS

- A key observation in probabilistic rough set model.
  - Three-way decisions are based on evaluation of an equivalence class with respect to a concept.
- We utilize a similar idea but with different evaluations based on multiple criteria.
- A two-player game is considered for each equivalence class.

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Formulation of Three-way Decisions with GTRS

- Game details
  - Players: They reflect different criteria used in evaluating a particular equivalence class. For example gini index, entropy.
  - Strategies: Two strategies are considered.
    - $s_1 = P$, is desired when the evaluation result with a criterion are above certain expectation.
    - $s_2 = N$, is desired when the evaluation result with a criterion is below certain expectation.
  - Payoff Functions: Defined in terms of measures that are used to evaluate a criterion.
    - The criterion of uncertainty evaluated with Shannon entropy.

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Three-way Decisions with GTRS

- The game for three-way decision making

		C_2	
		P	N
C_1	P	$u_{C_1}(P,P), u_{C_2}(P,P)$	$u_{C_1}(P,N), u_{C_2}(P,N)$
	N	$u_{C_1}(N,P), u_{C_2}(N,P)$	$u_{C_1}(N,N), u_{C_2}(N,N)$

- The solution concept of Nash equilibrium is used to determine the game outcome.

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Three-way Decisions with GTRS

- Three possible outcomes of the game.
  - Both players select.

$$u_{C_1}(P,P) \geq u_{C_1}(N,P) \ \ \& \ \ u_{C_2}(P,P) \geq u_{C_1}(P,N). \quad (4)$$

  - Both Players reject.

$$u_{C_1}(N,N) \geq u_{C_1}(P,N) \ \& \ u_{C_2}(N,N) \geq u_{C_1}(N,P) \quad (5)$$

  - One of the players select.

$$u_{C_1}(P,N) \geq u_{C_1}(N,N) \ \ \& \ \ u_{C_2}(P,N) \geq u_{C_1}(P,P) \quad (6)$$
$$u_{C_1}(N,P) \geq u_{C_1}(P,P) \ \ \& \ \ u_{C_2}(N,P) \geq u_{C_1}(N,N) \quad (7)$$

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Three-way Decisions with GTRS

- Three-way decisions are made according to the following rules.

$$\begin{aligned} \text{Acceptance:} \quad & \text{if} \quad u_{C_1}(P,P) \geq u_{C_1}(P,N) \ \ \& \\ & \qquad u_{C_2}(P,P) \geq u_{C_2}(N,P), \\ \text{Rejection:} \quad & \text{if} \quad u_{C_1}(N,N) \geq u_{C_1}(N,P) \ \ \& \\ & \qquad u_{C_2}(N,N) \geq u_{C_2}(P,N), \\ \text{Deferment:} \quad & \quad \textit{otherwise} \end{aligned} \tag{8}$$

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Conclusion

- The probabilistic rough sets uses a single criterion of conditional probability to obtain three-way decisions.
- The game-theoretic rough set model can incorporate multiple criteria to obtain three-way decisions.
- The consideration of multiple criteria may allow for more informed and flexible decisions.

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Questions?