Application of Game-theoretic Rough Sets in Recommender Systems

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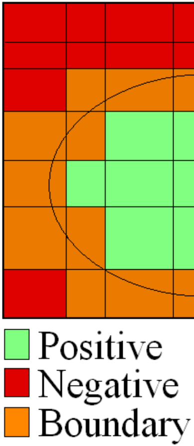
Rough Sets

- Proposed by Professor Zdzislaw Pawlak in the early 1980s.
- The basic idea is to approximate a concept C by using,
  - Lower approximation given by $\underline{apr}(C)\{x \in U|[x] \subseteq C\}$,
  - Upper approximation given by $\overline{apr}(C)\{x \in U|[x] \bigcap C \neq \phi\}$.
- Three regions may be defined using these approximations.
  - $POS(C) = \underline{apr}(C)$.
  - $BND(C) = \overline{apr}(C) - \underline{apr}(C)$.
  - $NEG(C) = U - (POS(C) \bigcup BND(C))$.

The Three Regions in Rough Sets



Probabilistic Rough Sets

- Introducing probabilities to define the rough set based approximations with a pair of (α, β) thresholds (Yao, 2008).
  - The (α, β) probabilistic approximations are given by,

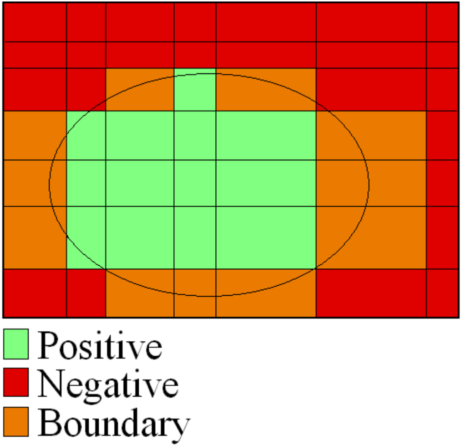
$$\begin{aligned} \underline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{x \in U \mid Pr(C|[x]) \geq \alpha\}, \\ \overline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{x \in U \mid Pr(C|[x]) > \beta\}. \end{aligned} \quad (1)$$

  - Probabilistic positive, negative and boundary regions:

$$\begin{aligned} \mathrm{POS}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \geq \alpha\}, \\ \mathrm{NEG}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \leq \beta\}, \\ \mathrm{BND}_{(\alpha,\beta)}(C) &= \{x \in U \mid \beta < Pr(C|[x]) < \alpha\}. \end{aligned} \quad (2)$$

Yao, Y. Y., (2008). Probabilistic rough set approximations, International Journal of Approximate Reasoning, 49.

The Three Regions in Probabilistic Rough Sets



Probabilistic Rough Sets: A Main Result and Key Issue

- A main result of probabilistic rough sets is that the three regions are controlled and defined by a pair of thresholds.

$$\begin{aligned} \text{POS:} &\quad \text{if } P(C|[x]) \geq \alpha, \\ \text{NEG:} &\quad \text{if } P(C|[x]) \leq \beta, \text{ and} \\ \text{BND:} &\quad \text{if } \beta < P(C|[x]) < \alpha. \end{aligned} \tag{3}$$

- A major difficulty is the interpretation and determination of the (α, β) thresholds (Yao, 2011).

Yao, Y.Y., (2011). Two semantic issues in a probabilistic rough set model. Fundamenta Informaticae 108(3).

Determination of (α, β) Probabilistic Thresholds

- The determination of thresholds is generally approached as an optimization of some property or examining a tradeoff solution between multiple criteria.
- Some recent notable attempts include,
  - Optimization viewpoint (Jia et al., 2011),
  - Multi-view model(Li and Zhou, 2011),
  - Method using probabilistic model criteria (Liu et al., 2011),
  - Information-theoretic interpretation (Deng and Yao, 2012) ,
  - Game-theoretic framework (Herbert and Yao, 2011).
- We consider the game-theoretic rough set model.

Jia, X. Y., Li, W. W., Shang, L., Chen, J. J., (2011). An optimization viewpoint of DTRS model. In: (RSKT'11).
Li, H.X., Zhou, X.Z., (2011). Risk decision making based on DTRS... IJCIS 4.
Liu, D., Li, T.R., Ruan, D., (2011). Probabilistic model criteria with DTRS. Information Science 181.
Deng, X. F., Yao, Y. Y., (2012). An information-theoretic interpretation of thresholds in PRS. In: (RSCTC'12).
Herbert, J.P., Yao, J.T., 2011. Game-theoretic rough sets. Fundamenta Informaticae 108(3-4).

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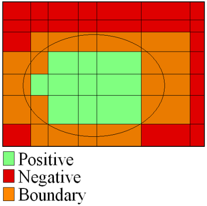
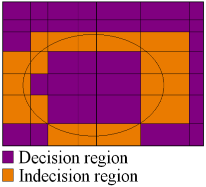
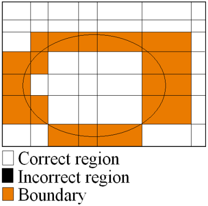
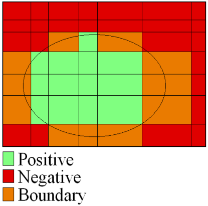
Motivation: How GTRS can Help

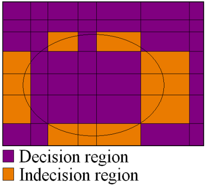


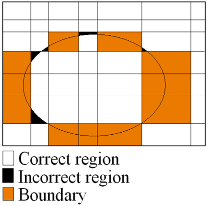




Pawlak model







Probabilistic Model

Motivation: How GTRS can Help

- The Pawlak positive and negative regions are completely accurate.
  - However, we may not be able to make decisions for majority of the objects.
- The probabilistic model allows for making more decisions by expanding the positive and negative regions.
  - This however leads to some errors.
- The thresholds (α, β) control the tradeoff between accuracy and generality in the probabilistic rough set model.

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Game Theory

- Game theory is a core subject in decision sciences.
- The basic game components include.
  - Players.
  - Strategies.
  - Payoffs.
- A classical example in Game Theory: The prisoners dilemma.

		p_2	
		confess	don't confess
p_1	confess	p_1 serves 10 year, p_2 serves 10 years	p_1 serves 0 year p_2 serves 20 years
	don't confess	p_1 serves 20 year, p_2 serves 0 years	p_1 serves 1 year, p_2 serves 1 years

A Formal Game Definition

- A game may be formally defined as a tuple $\{P, S, u\}$ (Brown and Shoham, 2008),
  - P is a finite set of n players, indexed by i,
  - $S = S_1 \times ... \times S_n$, where S_i is a finite set of strategies available to player i. Each $s = (s_1, s_2, ..., s_n) \in S$ is called a strategy profile where player i selects strategy s_i.
  - $u = (u_1, ..., u_n)$ where $u_i : S \longmapsto \Re$ is a real-valued utility or payoff function for player i.

Brown L. K. and Shoham, Y., (2008). Essentials of Game Theory: A Concise Multidisciplinary Introduction.

Game-theoretic Rough Sets

- Utilizing a game-theoretic setting for analyzing rough sets.
- Determining the probabilistic thresholds to obtain the three regions and the implied three-way decisions.
- Current GTRS formulations considering different players and utility functions to determine thresholds.
    - Improving classification ability (Herbert and Yao, 2011).
    - Effective rule mining (Azam and Yao, 2012).
    - Reducing region uncertainties (Yao and Azam, 2013; 2014).
    - Optimizing Gini Coefficient (Yan, 2013).
    - Optimizing accuracy and generality (Azam and Yao, 2014).

Herbert, J.P., Yao, J.T. (2011). Game-theoretic rough sets. Fundamenta Informaticae 108(3-4).
Azam, N., Yao, J. T., (2012). Multiple criteria decision analysis with GTRS. In: (RSKT'12).
Azam, N., Yao, J. T., (2013). Analyzing uncertainties of probabilistic rough set regions with GTRS. IJAR.
Azam, N., Yao, J. T., (2013). GTRS for Recommender Systems KBS.
Yan, Z., (2013). Optimizing GINI coefficient of probabilistic rough set regions using GTRS. In: (CCECE'13).
J.T. Yao, N. Azam. (2014). Web-based Medical Decision Support Systems for Three-way Medical Decision Making with GTRS, IEEE TFS.

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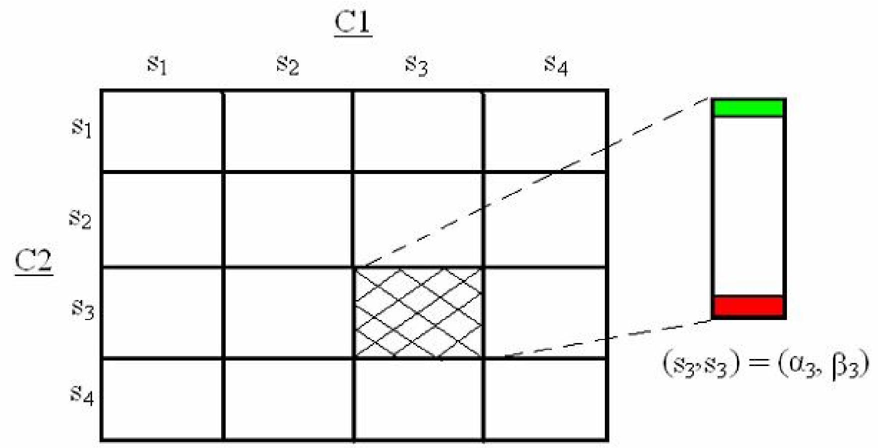
An Intuitive Representation of GTRS



Recommender Systems

- Provide useful guidance to users in decisions related to personal taste and choice.
  - Require some sort of intelligent mechanism to make recommendations.
- We examine two properties of recommendation predictions with GTRS.
- Accuracy
  - How close a recommender system predictions are to the actual user preferences.
- Generality.
  - The relative number of users for whom we actually make recommendation predictions.
- Highly accurate recommendations may not be possible for majority of the users.

Properties of Accuracy and Generality

$$
\begin{aligned}
Accuracy(\alpha, \beta) &= \frac{\text{Correctly classified objects by } \mathrm{POS}_{(\alpha,\beta)} \text{and } \mathrm{NEG}_{(\alpha,\beta)}}{\text{Total classified objects by } \mathrm{POS}_{(\alpha,\beta)} \text{and } \mathrm{NEG}_{(\alpha,\beta)}}, \\
&= \frac{|(\mathrm{POS}_{(\alpha,\beta)} \cap C) \bigcup (\mathrm{NEG}_{(\alpha,\beta)} \cap C^c)|}{|\mathrm{POS}_{(\alpha,\beta)} \bigcup \mathrm{NEG}_{(\alpha,\beta)}|}, \\
Generality(\alpha, \beta) &= \frac{\text{Total classified objects by } \mathrm{POS}_{(\alpha,\beta)} \text{and } \mathrm{NEG}_{(\alpha,\beta)}}{\text{Number of objects in } U}. \\
&= \frac{|\ \mathrm{POS}_{(\alpha,\beta)} \bigcup \mathrm{NEG}_{(\alpha,\beta)}|}{|U|}.
\end{aligned}
$$

Example of Accuracy and Generality Tradeoff

	Movie 1	Movie 2	Movie 3	Movie 4
U_1	+	+	+	+
U_2	+	+	-	+
U_3	+	-	+	+
U_4	-	+	+	+
U_5	+	+	-	+
U_6	-	+	+	-
U_7	-	+	-	+
U_8	-	-	+	+
U_9	-	+	+	+
	...	...	...	...
	...	...	...	...
	...	...	...	...
U_{23}	-	-	+	-
U_{24}	+	-	-	-
U_{25}	-	-	-	-
U_{26}	+	-	-	-

Using the Probabilistic Rough Set Framework

- The concept of interest in this case is to determine the positive ratings for the Movie 4, i.e., Movie 4 $= +$.
- We approximate this concept in the probabilistic rough sets framework.
- The following equivalence classes are created based on the data.

$$
\begin{array}{ll}
X_1 = \{U_1\}, & X_2 = \{U_2, U_5\}, \\
X_3 = \{U_3, U_{10}, U_{15}, U_{16}, U_{20}\}, & X_4 = \{U_4, U_6, U_9, U_{11}\}, \\
X_5 = \{U_7, U_{17}, U_{21}\}, & X_6 = \{U_8, U_{22}, U_{23}\}, \\
X_7 = \{U_{12}, U_{13}, U_{18}, U_{24}, U_{26}\}, & X_8 = \{U_{14}, U_{19}, U_{25}\}.
\end{array}
$$

Using the Probabilistic Rough Set Framework

- The conditional probability $P(C|X_i)$ for equivalence classes $X_1, ..., X_8$ are 1.0, 1.0, 0.8, 0.75, 0.67, 0.33, 0.2 and 0.0, respectively.
- The probability $P(X_i)$ for equivalence classes $X_1, ..., X_8$ are determined as 0.038, 0.077, 0.192, 0.154, 0.115, 0.115, 0.192 and 0.115, respectively.
- The accuracy and generality of the Pawlak model may be calculated based on this information.

Accuracy and Generality of the Pawlak Model

$$
\begin{aligned}
Accuracy(\alpha,\beta) &= \frac{|((X_1 \bigcup X_2) \cap C) \bigcup (X_8 \cap C^c)|}{|X_1 \bigcup X_2 \bigcup X_8|}, \\
&= \frac{|\{U_1, U_2, U_5, U_{14}, U_{19}, U_{25}\}|}{|\{U_1, U_2, U_5, U_{14}, U_{19}, U_{25}\}|} = \frac{6}{6} = 1.0, \\
Generality(\alpha,\beta) &= \frac{|(X_1 \bigcup X_2 \bigcup X_8)|}{|U|}, \\
&= \frac{\{U_1, U_2, U_5, U_{14}, U_{19}, U_{25}\}}{|U_1, U_2, ..., U_{27}|} = \frac{6}{26} = 0.2307.
\end{aligned}
$$

- 100% accurate recommendations for 23.07% of the users.
- More recommendations may be possible if we lower our exception of being 100% accurate.
- Accuracy versus generality $\rightarrow$ tradeoff perspective.

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Accuracy versus Generality Game

- The players in the game: Accuracy vs. Generality.
- The Strategies.
  - Considering an initial thresholds of $(\alpha, \beta) = (1, 0)$.
  - Three strategies for the players are formulated.
  - $s_1 = \alpha_{\downarrow}$ = decrease α.
  - $s_2 = \beta_{\uparrow}$ = increase β.
  - $s_3 = \alpha_{\downarrow}\beta_{\uparrow}$ = decrease α and increase β.
  - We consider an increase or decrease of 25% in this example.

The Game in a Payoff Table

		Gen.		
		$s_1 = \alpha_\downarrow$ = 25% dec. α	$s_2 = \beta_\uparrow$ = 25% inc. β	$s_3 = \alpha_\downarrow\beta_\uparrow$ = 25% (dec. α & inc. β)
Acc.	$s_1 = \alpha_\downarrow$ = 25% dec. α	(0.83,0.69)	(0.85,0.77)	(0.83,0.89)
	$s_2 = \beta_\uparrow$ = 25% inc. β	(0.85,0.77)	(0.86,0.54)	**(0.83,0.89)**
	$s_3 = \alpha_\downarrow\beta_\uparrow$ = 25% (dec. α & inc. β)	(0.83,0.89)	(0.83,0.89)	(0.8077,0.1.0)

- The cell with bold values is the Nash solution (Nash, 2007).
- We can make recommendations for 89% users which are 83% accurate.

Nash, J., 1950. Non-cooperative games. Annals of mathematics.

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Experimental Setup

- We consider the *movielen* dataset.
- The dataset consists of three different tables,
  - User table: demographic information about the 6,040 users.
  - Movie table: information about 3,952 movies.
  - Ratings table: 1 million user ratings on a 5-star scale.
- We considered the ratings of about 400 users which resulted in 58,000 ratings.
- The ratings were simplified in a ("like", "dislike") model.
- Two sets of experiments were conducted.
  - Task 1: Ratings of 4 or 5 indicate "like".
  - Task 2: Rating of 5 indicate "like".
- Only top 10 most frequently rated movies are considered.

Repeated Game

- It may be difficult to find effective values for the thresholds in a single game.
- Considering a repeated game.
- The output of one game may be used as input for the next game.
- The following stop conditions are used to stop the repeated game.
  - Boundary region becomes empty, or
  - The positive region size exceeds the prior probability of the concept C, or
  - *Generality*(α, β) exceeds *Accuracy*(α, β).

Experimental Results: Collaborative Filtering

- Train results for data with Task 1.

Prediction for Movie	Accuracy		Generality	
	GTRS	Pawlak	GTRS	Pawlak
1.	0.9140	**1.0**	**0.9540**	0.6481
2.	0.9525	**1.0**	**0.9198**	0.6878
3.	0.9829	**1.0**	**0.9782**	0.8584
4.	0.9712	**1.0**	**0.9546**	0.8037
5.	0.9605	**1.0**	**0.9594**	0.7798
6.	0.9739	**1.0**	**0.9271**	0.7916
7.	0.9792	**1.0**	**0.9875**	0.9271
8.	0.9615	**1.0**	**0.9314**	0.7512
9.	0.9766	**1.0**	**0.9625**	0.8633
10.	0.9687	**1.0**	**0.9825**	0.8641
Average	0.9641	**1.0**	**0.9557**	0.7975

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Experimental Results: Collaborative Filtering

- Test results for data with Task 1.

Prediction for Movie	Accuracy		Generality	
	GTRS	Pawlak	GTRS	Pawlak
1.	**0.4898**	0.4448	**0.8965**	0.6847
2.	0.6425	**0.6426**	**0.8977**	0.7371
3.	**0.5873**	0.5484	**0.9713**	0.8730
4.	**0.5950**	0.5749	**0.9537**	0.8256
5.	**0.5802**	0.5659	**0.9143**	0.8083
6.	**0.6348**	0.6126	**0.9016**	0.8145
7.	**0.6680**	0.6598	**0.9838**	0.9627
8.	**0.6407**	0.6303	**0.9102**	0.7807
9.	0.6194	**0.6269**	**0.9428**	0.8827
10.	0.7252	**0.7344**	**0.9750**	0.9102
Average	**0.6183**	0.6041	**0.9347**	0.8279

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Experimental Results: Collaborative Filtering

- Train results for data with Task 2.

Prediction for Movie	Accuracy		Generality	
	GTRS	Pawlak	GTRS	Pawlak
1.	0.9375	**1.0**	**0.9678**	0.7529
2.	0.9818	**1.0**	**0.9346**	0.8248
3.	0.9807	**1.0**	**0.9548**	0.8397
4.	0.9795	**1.0**	**0.9596**	0.8251
5.	0.9763	**1.0**	**0.9836**	0.8829
6.	0.9799	**1.0**	**0.9522**	0.8430
7.	0.9865	**1.0**	**0.9951**	0.9572
8.	0.9756	**1.0**	**0.9519**	0.8493
9.	0.9828	**1.0**	**0.9750**	0.9008
10.	0.9782	**1.0**	**1.0**	0.9129
Average	0.9759	**1.0**	**0.9675**	0.8589

Experimental Results: Collaborative Filtering

- Test results for data with Task 2.

Prediction for Movie	Accuracy		Generality	
	GTRS	Pawlak	GTRS	Pawlak
1.	**0.6488**	0.5962	**0.9428**	0.7835
2.	**0.7613**	0.7382	**0.9303**	0.8418
3.	**0.7372**	0.7147	**0.9526**	0.8617
4.	**0.6958**	0.6631	**0.9451**	0.8444
5.	**0.7092**	0.6877	**0.9701**	0.9017
6.	**0.7684**	0.7521	**0.9477**	0.8706
7.	**0.7596**	0.7554	**0.9875**	0.9726
8.	**0.7676**	0.7444	**0.9452**	0.8655
9.	**0.7535**	0.7436	**0.9775**	0.9327
10.	**0.8201**	0.8131	**0.9900**	0.9415
Average	**0.7421**	0.7208	**0.9589**	0.8816

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Discussion

- The GTRS always lead to better generality compared to the Pawlak model.
- The GTRS performance is better in 7 out of 10 movies on the testing data of Task 1.
- The GTRS outperforms the Pawlak model in all aspects on the testing data of Task 2.

Conclusions

- We examined the properties of accuracy and generality of recommendations.
- Making highly accurate recommendations for majority of the users is not always possible.
  - One has to consider tradeoff between accuracy and generality which is controlled by thresholds (α, β) in the probabilistic rough sets.
- The role of GTRS is considered for determining the thresholds.
- Experimental results suggests that the GTRS improve the accuracy and generality compared to the Pawlak model.

Acknowledgment

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Questions?

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