Formulating Game Strategies in Game-theoretic Rough Sets

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Probabilistic Rough Sets

- Introducing probabilities to define the rough set based approximations with a pair of (α, β) thresholds (Yao, 2008).
  - The (α, β) probabilistic approximations are given by,

$$\begin{aligned} \underline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{[x] \in U/E \mid Pr(C|[x]) \geq \alpha\}, \\ \overline{apr}_{(\alpha,\beta)}(C) &= \bigcup\{[x] \in U/E \mid Pr(C|[x]) > \beta\}. \end{aligned} \quad (1)$$

  - Probabilistic positive, negative and boundary regions:

$$\begin{aligned} \mathrm{POS}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \geq \alpha\}, \\ \mathrm{NEG}_{(\alpha,\beta)}(C) &= \{x \in U \mid Pr(C|[x]) \leq \beta\}, \\ \mathrm{BND}_{(\alpha,\beta)}(C) &= \{x \in U \mid \beta < Pr(C|[x]) < \alpha\}. \end{aligned} \quad (2)$$

Yao, Y. Y., (2008). Probabilistic rough set approximations, International Journal of Approximate Reasoning, 49.

Probabilistic Rough Sets: A Main Result and Key Issue

- A main result of probabilistic rough sets is that the rules for determining the three regions are given by,

$$\begin{aligned} &\text{Acceptance:} && \text{if } P(C|[x]) \geq \alpha, \\ &\text{Rejection:} && \text{if } P(C|[x]) \leq \beta, \text{ and} \\ &\text{Deferment:} && \text{if } \beta < P(C|[x]) < \alpha. \end{aligned} \quad (3)$$

- A major difficulty is the interpretation and determination of the (α, β) thresholds (Yao, 2011).

Yao, Y.Y., (2011). Two semantic issues in a probabilistic rough set model. Fundamenta Informaticae 108(3).

Determination of (α, β) Probabilistic Thresholds

- The determination of probabilistic thresholds can generally be approached as an optimization problem based on criterion C (Deng and Yao, 2012).

$$\arg \min_{(\alpha,\beta)} C(\alpha, \beta), \text{ where}$$

$$C(\alpha, \beta) = C_P(\alpha, \beta) + C_N(\alpha, \beta) + C_B(\alpha, \beta). \qquad (4)$$

Deng, X. F., Yao, Y. Y., (2012). An information-theoretic interpretation of thresholds in PRS. In: (RSCTC'12).

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Attempts for Determination of Probabilistic Thresholds

- Recent attempts.
  - Optimization viewpoint (Jia et al., 2011),
  - Multi-view model(Li and Zhou, 2011),
  - Method using probabilistic model criteria (Liu et al., 2011),
  - Information-theoretic interpretation (Deng and Yao, 2012) ,
  - Game-theoretic framework (Herbert and Yao, 2011).
- We consider the game-theoretic rough set model.

Jia, X. Y., Li, W. W., Shang, L., Chen, J. J., (2011). An optimization viewpoint of DTRS model. In: (RSKT'11).
Li, H.X., Zhou, X.Z., (2011). Risk decision making based on DTRS... IJCIS 4.
Liu, D., Li, T.R., Ruan, D., (2011). Probabilistic model criteria with DTRS. Information Science 181.
Deng, X. F., Yao, Y. Y., (2012). An information-theoretic interpretation of thresholds in PRS. In: (RSCTC'12).
Herbert, J.P., Yao, J.T., 2011. Game-theoretic rough sets. Fundamenta Informaticae 108(3-4).

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Game Theory

- Game theory is a core subject in decision sciences.
- The basic game components include.
  - Players.
  - Strategies.
  - Payoffs.
- A classical example in Game Theory: The prisoners dilemma.

		p_2	
		confess	don't confess
p_1	confess	p_1 serves 10 year, p_2 serves 10 years	p_1 serves 0 year p_2 serves 20 years
	don't confess	p_1 serves 20 year, p_2 serves 0 years	p_1 serves 1 year, p_2 serves 1 years

A Formal Game Definition

- A game may be formally defined as a tuple $\{P, S, u\}$ (Brown and Shoham, 2008),
  - P is a finite set of n players, indexed by i,
  - $S = S_1 \times ... \times S_n$, where S_i is a finite set of strategies available to player i. Each $s = (s_1, s_2, ..., s_n) \in S$ is called a strategy profile where player i selects strategy s_i.
  - $u = (u_1, ..., u_n)$ where $u_i : S \longmapsto \Re$ is a real-valued utility or payoff function for player i.

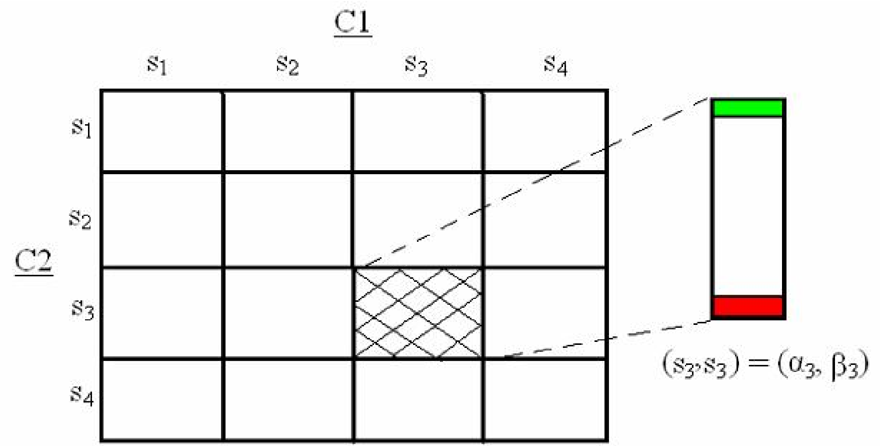
Brown L. K. and Shoham, Y., (2008). Essentials of Game Theory: A Concise Multidisciplinary Introduction.

Game-theoretic Rough Sets

- Utilizing a game-theoretic setting for analyzing rough sets.
- Determining the probabilistic thresholds to obtain the three regions and the implied three-way decisions.
- Current GTRS formulations considering different players and utility functions to determine thresholds.
  - Improving classification ability (Herbert and Yao, 2011).
  - Effective rule mining (Azam and Yao, 2012).
  - Reducing region uncertainties (Azam and Yao, 2013).
  - Optimizing Gini Coefficient (Yan, 2011).

Herbert, J.P., Yao, J.T. (2011). Game-theoretic rough sets. Fundamenta Informaticae 108(3-4).
Azam, N., Yao, J. T., (2012). Multiple criteria decision analysis with GTRS. In: (RSKT'12).
Azam, N., Yao, J. T., (2013). Analyzing uncertainties of probabilistic rough set regions with GTRS. IJAR
Yan, Z., (2013). Optimizing GINI coefficient of probabilistic rough set regions using GTRS. In: (CCECE'13).

Game-theoretic Rough Sets



Strategies in Existing GTRS based Approaches

- The strategies in existing approaches are generally formulated based on an initial threshold configuration $(\alpha, \beta) = (1, 0)$.
    - This only allows the strategies to be formulated in the form of decreasing levels for α and increasing levels for β.
- This provides limited opportunities for fine tunning the thresholds.
- Introducing additional approaches in the aim of determining more effective thresholds.
    - Making use of different initial conditions that leads to different strategy formulation approaches.

The Two Ends Approach

- This approach is generally used in GTRS (Herbert and Yao, 2011; Azam and Yao, 2012).
- Considers an initial threshold configuration of $(\alpha, \beta) = (1, 0)$.
  - Strategies are formulated as decreasing levels for α and increasing levels for β.
- We call it two ends approach as the thresholds are modified from the two extreme ends.

Herbert, J.P., Yao, J.T. (2011). Game-theoretic rough sets. Fundamenta Informaticae 108(3-4).
Azam, N., Yao, J. T., (2012). Multiple criteria decision analysis with GTRS. In: (RSKT'12).
Azam, N., Yao, J. T., (2013). Analyzing uncertainties of probabilistic rough set regions with GTRS. IJAR

A Game for the Two Ends Approach

- An example game based on the two ends approach.

		P_2		
		$s_1 = \alpha_\downarrow$	$s_2 = \beta_\uparrow$	$s_3 = \alpha_\downarrow\beta_\uparrow$
P_1	$s_1 = \alpha_\downarrow$	....	.....	.....
	$s_2 = \beta_\uparrow$	....	.....	.....
	$s_3 = \alpha_\downarrow\beta_\uparrow$	....	.....	.....

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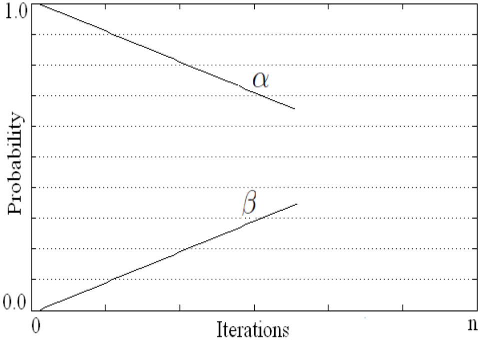
Threshold Modification Trend with the Approach

- When a repeated game based on the approach is considered, the thresholds are expected to be modified as in the figure.



When to Use the Approach ?

- Considering data of high quality where the classes or concepts are well defined.
- A minimum size boundary is expected in this case.
  - Accurate decisions can be made with high rate while keeping α close to 1.0 and β to 0.0.
  - Minor adjustments in thresholds $(\alpha, \beta) = (1.0)$ may be considered for obtaining an effective probabilistic model.
- This approach can provide quick converge in such situations.

The Middle Approach

- Considers an initial threshold configuration of $\alpha = \beta$.
  - Considering $\beta < \alpha$, the strategies are formulated as increasing levels for α and decreasing levels for β.
- We call it the middle approach as the thresholds are modified from a common middle value.

A Game for the Middle Approach

- An example game based on the middle approach.

		P_2		
		$s_1 = \alpha_\uparrow$	$s_2 = \beta_\downarrow$	$s_3 = \alpha_\uparrow\beta_\downarrow$
	$s_1 = \alpha_\uparrow$	....	.....	....
P_1	$s_2 = \beta_\downarrow$	....	.....	....
	$s_3 = \alpha_\uparrow\beta_\downarrow$	....	.....	....

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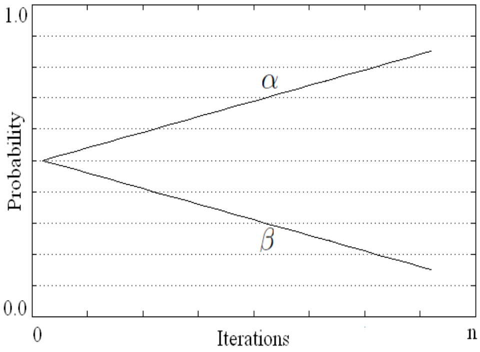
Threshold Modification Trend with the Approach

- When a repeated game based on the approach is considered, the thresholds are expected to be modified as in the figure.



When to Use the Approach ?

- Considering data of very low quality with high uncertainty.
- We expect many objects in the boundary region leading to its larger size.
  - The number of available certain decisions are very limited.
- This approach can be useful in such situations.
  - Starting from a zero sized boundary.
  - Carefully increase the boundary until desired performance levels are reached based on certain criterion.

The Random Approach

- Considering random values for starting the threshold configuration.
  - Assuming that we do not know the modification direction that will provide effective threshold values.
  - The formulated strategies should therefore provide options for both increasing or decreasing a particular threshold.
- The strategies will allow to investigate threshold values in the neighbourhood of the starting point.

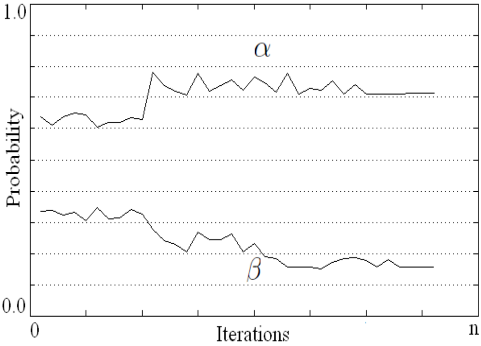
A Game for the Random Approach

- An example game based on the random approach.

		P_2	
		$s_1 = \alpha_\uparrow$	$s_2 = \alpha_\downarrow$
P_1	$s_1 = \beta_\uparrow$	....	.....
	$s_2 = \beta_\downarrow$	....	.....

Threshold Modification Trend with the Approach

- When a repeated game based on the approach is considered, the thresholds are expected to be modified as in the figure.



When to Use the Approach ?

- Considering data of with intermediate level of uncertainty.
- The effective threshold values can be located anywhere in the threshold space.
- This approach can play a role in determining thresholds in such situations.

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The Range Approach

- The strategies may also be formulated by considering a possible range for the thresholds.
  - Evaluating the entire range may be not be feasible.
  - Selected values in the range may be evaluated.
  - For instance, considering an initial range for α as [0.5, 1.0], the strategies $s_1 = \alpha_1, s_2 = \alpha_2, ..., s_n = \alpha_n$ may represent different values in the range, with conditions such as $0.5 \leq \alpha_1 < \alpha_2 <, ..., < \alpha_n \leq 1.0$.
  - The game may start from a wider range which is iteratively reduced.

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A Game for the Range Approach

- An example game based on the range approach.

		P_2		
		$s_1 = \alpha_1$	.....	$s_n = \alpha_n$
P_1	$s_1 = \beta_1$	....	.....	....
	...	....	.....	....
	$s_n = \beta_n$	....	.....	....

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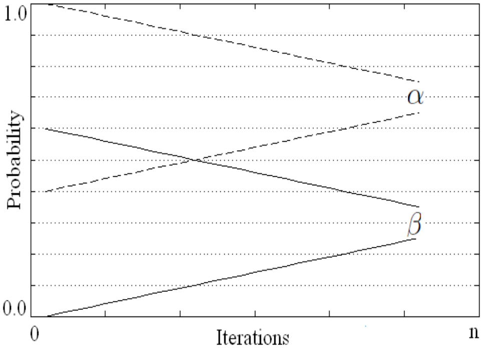
Threshold Modification Trend with the Approach

- When a repeated game based on the approach is considered, the thresholds are expected to be modified as in the figure.



When to Use the Approach ?

- The approach may be useful under tight computing constraint.
- Quick determination may be achieved when the range is drastically reduced at each iteration.

Threshold Configuration with the Two Ends Approach

- Considering a game between the properties of accuracy and generality of the rough set model.
- A general model may not be necessarily accurate and vice versa.
- The aim is to find a balance between the two properties.

$$\mathit{Accuracy}(\alpha,\beta) = \frac{\text{Correctly classified objects by } \mathrm{POS}_{(\alpha,\beta)} \text{ and } \mathrm{NEG}_{(\alpha,\beta)}}{\text{Total classified objects by } \mathrm{POS}_{(\alpha,\beta)} \text{ and } \mathrm{NEG}_{(\alpha,\beta)}},$$

$$\mathit{Generality}(\alpha,\beta) = \frac{\text{Total classified objects by } \mathrm{POS}_{(\alpha,\beta)} \text{ and } \mathrm{NEG}_{(\alpha,\beta)}}{\text{Number of objects in } U}.$$

Probabilistic Information about a Concept

- Considering the following probabilistic information about a concept C with respect to 18 equivalence classes.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
$Pr(X_i)$	0.034	0.099	0.132	0.017	0.068	0.017	0.056	0.049	0.049
$Pr(C\|X_i)$	1.0	0.96	0.91	0.86	0.81	0.77	0.71	0.64	0.53

	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
$Pr(X_i)$	0.115	0.072	0.01	0.119	0.019	0.042	0.009	0.047	0.046
$Pr(C\|X_i)$	0.49	0.43	0.38	0.31	0.27	0.22	0.15	0.09	0.02

A Game based on the Two Ends Approach

- The players in the game: Accuracy vs. Generality.
- The Strategies.
  - Considering an initial thresholds of $(\alpha, \beta) = (1, 0)$.
  - Three strategies for the players are formulated.
  - $s_1 = \alpha_{\downarrow} =$ decrease α by 5%.
  - $s_2 = \beta_{\uparrow} =$ increase β by 5%.
  - $s_3 = \alpha_{\downarrow}\beta_{\uparrow} =$ decrease α by 5% and increase β by 5%.

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Payoff Table for the Game

- Considering initial thresholds $(\alpha, \beta) = (1, 0)$.

		Gen.		
		$s_1 = \alpha_\downarrow$ = 5% dec. α	$s_2 = \beta_\uparrow$ = 5% inc. β	$s_3 = \alpha_\downarrow\beta_\uparrow$ = 5% (dec. α & inc. β)
Acc.	$s_1 = \alpha_\downarrow$ = 5% dec. α	(0.941,0.265)	(0.937,0.179)	(0.946,0.311)
	$s_2 = \beta_\uparrow$ = 5% inc. β	(0.973,0.179)	(0.959,0.127)	**(0.959,0.226)**
	$s_3 = \alpha_\downarrow\beta_\uparrow$ = 5% (dec. α & inc. β)	(0.946,0.311)	(0.959,0.226)	(0.941,0.358)

- The thresholds determined by the game using the game solution in the form the of Nash equilibrium are $(0.95, 0.1)$.

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Repeating the Game

- Next considering initial thresholds $(\alpha, \beta) = (0.95, 0.10)$.

		Gen.		
		$s_1 = \alpha_{\downarrow}$ = 5% dec. α	$s_2 = \beta_{\uparrow}$ = 5% inc. β	$s_3 = \alpha_{\downarrow}\beta_{\uparrow}$ = 5% (dec. α & inc. β)
Acc.	$s_1 = \alpha_{\downarrow}$ = 5% dec. α	(0.938,0.375)	(0.939,0.367)	(0.937,0.384)
	$s_2 = \beta_{\uparrow}$ = 5% inc. β	**(0.939,0.367)**	(0.956,0.235)	(0.939,0.367)
	$s_3 = \alpha_{\downarrow}\beta_{\uparrow}$ = 5% (dec. α & inc. β)	(0.937,0.384)	(0.939,0.367)	(0.936,0.384)

- The thresholds determined by the game using the game solution in the form the of Nash equilibrium are $(0.90, 0.15)$.

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Repeating the Game

- Next considering initial thresholds $(\alpha, \beta) = (0.90, 0.15)$.

		Gen.		
		$s_1 = \alpha_\downarrow$ = 5% dec. α	$s_2 = \beta_\uparrow$ = 5% inc. β	$s_3 = \alpha_\downarrow\beta_\uparrow$ = 5% (dec. α & inc. β)
Acc.	$s_1 = \alpha_\downarrow$ = 5% dec. α	(0.917,0.452)	(0.936,0.384)	(0.917,0.452)
	$s_2 = \beta_\uparrow$ = 5% inc. β	(0.937,0.384)	(0.923,0.409)	**(0.920,0.426)**
	$s_3 = \alpha_\downarrow\beta_\uparrow$ = 5% (dec. α & inc. β)	(0.917,0.452)	(0.920,0.426)	(0.905,0.494)

- The thresholds determined by the game using the game solution in the form the of Nash equilibrium are $(0.85, 0.25)$.

The Repetitive Game with the Approach

- The iterative game in this way resulted in the following threshold sequence.
  - $\alpha : 1.0 \rightarrow 0.95 \rightarrow 0.90 \rightarrow 0.85.$
  - $\beta : 0.0 \rightarrow 0.1 \rightarrow 0.15 \rightarrow 0.25.$
- The process may be stopped when an acceptable performance level is reached.

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Conclusion

- The GTRS has recently received some attention for determining thresholds.
- We examine additional approaches for formulating strategies in GTRS that are based on different initial conditions.
- Some of these approaches may be more appropriate when different types of data are considered.
- The incorporation of these approaches in existing GTRS based methods may lead to interesting results.

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Questions?